MEASURING SPONTANEOUS ACCESSIBILITY FOR ITERATIVE TRANSIT PLANNING

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ABSTRACT

Public transit planners rely on measurements of network performance to anticipate the impact of changes to a transportation system. Accessibility-based measurements emphasize how well a transportation system allows individuals to reach desired opportunities, rather than maximizing network properties such as capacity. This paper presents a measurement of accessibility for transit customers making unanticipated, spontaneous trips. Measuring this Spontaneous Accessibility was facilitated by developing an open-source software tool that can analyze a transit network throughout an entire day, over a complete municipal boundary or transit agency service area, at fine spatial granularity, and without some of the simplifying assumptions made by previous studies. The tool is used to study Spontaneous Accessibility within the city of Seattle over a one year period featuring the opening of a light rail extension and restructures of bus service. Studies of this nature require only limited data sources but produce precise results, and thus can be utilized to measure iterative refinement of the transit network. Furthermore, techniques from the discipline of information theory provide insight into ways to reduce the computational demands, giving planners the ability to consider more alternatives.

INTRODUCTION

Individuals with use of a car enjoy an ease of access to distant destinations that those without vehicles do not. This difference in accessibility is especially evident when making unanticipated trips, such as going to a grocery store to pick up a single item needed for a recipe, visiting an urgent care clinic, or responding to a family emergency. Those with vehicles can start their journeys immediately in response to the spontaneous need or desire, and proceed to their destination in the most direct way. Those limited to walking and taking transit can make relatively short trips spontaneously by walking, but for longer trips they must contend with the fact that public transit generally runs on a schedule, transit lines may be indirect, and transfers may be required to reach destinations. All of these factors constrain the ability to complete unanticipated trips in a timely fashion, making private vehicle ownership an attractive proposition.

In the city of Seattle, the number of households without access to a car has been growing over the last five years (*1*). For commuting trips originating within the Seattle city limits and terminating in the downtown Central Business District, transit mode share dominates the use of single occupancy motorized vehicles (*2*). In spite of this, Seattle's rate of households with at least one vehicle exceeds that of cities including Milwaukee, Detroit, and San Francisco (*3*). The contrast between these observations could indicate that the desire for private vehicle ownership in Seattle is not tied solely to its use in getting to and from work. Thus, transit planners at agencies tasked with supporting a trend of decreased car ownership may want to prioritize the network's capability to support trips beyond those representative of standard commuting patterns.

This paper examines the ability of a public transit system, specifically that which exists in the city of Seattle, to support unanticipated trips. It defines the capability of a transit network to allow such trips as its Spontaneous Accessibility and describes comparable measurements for it that differ from accessibility measurements presented previously in the literature. Creating this measurement was facilitated by developing an open-source software tool that measures the transit network in ways that mirror a customer making an unexpected transit trip. The tool must evaluate the network over a full day period, as the need to make a transit trip may occur at any time. It must consider journeys from many origins to many destinations. A large set of origins ensures that the analysis is relevant to people throughout the studied area. Since the destinations of unexpected trips are by definition unpredictable, the analysis considers all destinations on the map. Furthermore, the tool must not measure the network in an overly abstract way; it should not make approximations that result in allowing transit trips that are infeasible in actuality. These requirements, however, work in opposition to the desire for transit planners to compute this measurement whenever considering service changes. Slow computation would preclude Spontaneous Accessibility measurements from being a part of periodic service evaluation, limiting them to being long-range planning tools. In support of finding balance between these priorities, this paper makes use of techniques from information theory to quantify the amount of error introduced by simplifying the analysis. As a demonstration of these techniques, Spontaneous Accessibility measurements are used to show the extent to which a rider's ability to take unanticipated trips within Seattle changed over a one year period that included a light rail line extension and bus restructures.

LITERATURE REVIEW

Path Finding

Nearly any measurement of a transportation network requires an understanding of the time expended to reach destinations. In the context of public transit, this requires knowledge of the transit routes one will take to their destination. Dial (*4*) describes the pathfinder program, which converts stops and the scheduled duration between them to a graph and uses a tree solving algorithm to find minimal paths from starting points. The algorithm assumes that starting locations are transit hubs and that connections between multiple lines only occur where transit lines share a stop. No walking is accounted for in this model and the transfer wait time is always half the frequency of the transit lines servicing a stop. For modeling transit networks abstractly, these limitations may be acceptable.

For more precise modeling of the transit network that is available to riders, path finding algorithms must have a notion of absolute time. Tong and Richardson (*5*) implement a method of finding minimal paths by converting a set of transit stops and schedules into a format where the minimal paths can be found by Dijkstra's graph algorithm. A graph models the transit network, like in earlier approaches, but the algorithm keeps track of the current time, and changes the durations between stops based on that time. It also models walking between certain stops. Crisalli and Rosati (*6*) present DY-RT, that uses the schedules of a larger regional bus and rail network, but make performance measurements at the level of municipalities. The heuristic approach taken by Ayed et al. (*7*) allows larger and more detailed representations of transit networks, at the cost of absolute accuracy. Though the approaches vary in focus on urban or interurban travel, they provide a higher-accuracy model of the transit network.

Though many path finding solutions use graph-theoretic approaches, other strategies exist. RAPTOR, described by Delling et al. (*8*), is not based on Dijkstra's algorithm; instead it uses dynamic programming to find best paths. The authors use it on a complete transit map of London with over 20,000 stops. As a result of its non-graph theoretic construction, it more easily supports parallel processing and calculating best paths for times in a range.

Transit Network Measurement

Many large-scale transportation planning decisions make use of the Urban Transportation Modeling System, which uses the four-step model consisting of trip generation, trip distribution, mode choice, and traffic assignment (*9*). This approach assumes some geographic area will generate a number of trips based on its underlying characteristics. Destination points have some measurement of gravity by which trips are attracted to them. The nature of the transit network influences the path that will be taken, and thus the expected ridership of a route within the network can be approximated. In the context of transit planning, route changes are tested by exploring how they impact the riderships projected by the four-step model.

The four-step model involves considerable data aggregation; trips are formed out of the aggregate properties of origin and destination areas. Hägerstand (*10*) asserts that such an approach does not take into account the fact that any trip occurs because of an individual's desires and needs, not out of the natures of regions. Such reasoning informs approaches to transit network evaluations that focus on what Hanson (*11*) describes as "personal accessibility", the ability of a person to reach sites where activities occur, or opportunities, from their home. Counting these opportunities and weighting them by their distance creates an "accessibility index" whereby the personal accessibility provided by the transportation system can be contrasted by origin point. Handy and Niemeier (*12*) argue that there are many dimensions to accessibility measurement and, while there is no best approach, a study's goal informs proper choices to make. Geurs and van Wee (*13*) assert that accessibility measures can be useful for making transportation and land use decisions, provided that a set of criteria are met. They further contend that person-based accessibility measurements, those which focus on what an individual can access given their time and space constraints, are effective for evaluating transportation network changes, but complexity of calculation and large data requirements make them difficult to perform. Work by Bertolini et al. (*14*) uses the concept of an isochrone to determine which neighborhoods can reach job centers within a time frame. The study only considers trips using a single transit route. In spite of its limitations, it demonstrates numerically-measured accessibility as a viable transit planning tool.

Geographic Information Systems

Understanding the ability of an individual to access destinations via transit is hindered when the walking paths that a person can take are not accurately known. Nyerges (*15*) describes the use of Geographic Information Systems (GIS) in deducing an accurate representation of locations in Seattle that can reach public transportation lines within a fixed walking distance. Tribby and Zandbergen (*16*) evaluate new bus service by producing an index that measures change in time to reach a single destination given transit need. GIS enables accurately modeling the walking distance from individual homes to buses, as well as transfers between them. Averages are used for transit ride and wait times. Work by Mavoa et al. (*17*) has similar characteristics, but measures access to a variety of destination types, such as schools, from individual property parcels. Silva and Pinho (*18*) propose the Structured Activity Layer, which expresses the diversity of opportunities accessible to an individual as an index measurement. O'Sullivan et al. (*19*) use GIS as a way to produce more accurate isochrones, rather than an index. Though a similar set of assumptions is made about transit travel time, GIS enables measurement at a scale befitting an individual's interaction with a transit network.

Schedule Data

Accurate measurements of accessibility must incorporate schedules, since real transit vehicles neither move at a constant rate, nor arrive at stops after a rider waits exactly half the headway. Though schedule-based path finding algorithms have long existed, the lack of easily accessible schedule data limited applications. The Transit Accessibility Planning Analyst described by Lei and Church (*20*) creates isochrones that use a single starting time, schedule data from contemporary timetables, walking time calculations from GIS, and a Dijkstra-like path finding algorithm. The authors propose evaluating transit networks using these isochrones by starting at a set of origins, and summing the destinations reachable over a set of starting times, but do not suggest a mechanism of selecting either. The availability of transit schedules in General Transit Feed Specification format (GTFS) (*21*) enabled considerable expansion in accessibility studies that use full schedule data. Work by Anderson et al. (*22*) suggests that, for some transit stops, it is critical to consider all starting times in a day, as accessibility varies both within and between hours. This is employed in a study of access to jobs in the Twin Cities area by Owen and Levinson (*23*), wherein a RAPTOR-like algorithm is used to compute the number of jobs that can be reached from every bus station in the network, over a two-hour rush hour commute period. The study performs aggregation at the census block level; initial walks to transit stops use an estimated walking time based on the straight-line distance. The accessibility value of a resource is weighted by a decay function to make distant opportunities less appealing than close ones. Owen and Levinson (*24*) use an unlimited transfer model and accurate walking routes, with census block groups as the unit of spatial granularity for measuring access to jobs. The accessibility values that they find correlate with observed transit mode share, a testament to the feasibility of using advanced accessibility-based measurements as a

TABLE 1 Comparison of Transit Accessibility Studies

Work	Transit Model	Walking Model	Origins & Destinations	Time Frame	Measurement
Bertolini et al.	Unimodal, avg. speed	N/A	Neighborhoods, job centers	N/A	Population, jobs
Tribbly and Zandbergen	Avg. ride time, wait time	Paths	Residences, single point	Range	Index: Time change given transit need
Mavoa et al.	Avg ride time, fixed wait time	Paths	Land parcels, various destination points	Range	Index: Time thresholds to destinations
Silva and Pinho	Avg. speed	Unspecified	Census tract	N/A	Index: variety of activities
Lei and Church	Full schedule	Paths	Single point, unspecified destinations	Single time	Reachable area
Owen and Levinson 2012	Full schedule	Straight line	Census block	Range	Time-weighted access to jobs
Owen and Levinson 2015	Full schedule	Paths	Census block group	Range	Access to jobs
Blanchard and Waddell	Avg ride time, wait time	Paths	Census block	Range	Access to jobs
Gillespie and Fahrenwald	Unspecified	Paths	Grid (.5 x .5 mi.)	Range	Time-weighted access to jobs
Conway et al.	Full schedule	Paths	Grid (78 x 78 m.)	Range	Access to jobs
Laquidara	Full schedule	Paths, elevation	Grid (176 x 281 m.)	Full day	Reachable area

transit planning technique. The UrbanAccess tool, proposed by Blanchard and Waddell (*25*), uses similar data sources, but takes the approach of averaging headway times for a time period rather than computing reachability at every minute. Gillespie and Fahrenwald (*26*) add the dimension of automobile travel to transit to produce a time-weighted access to jobs measurement. It is notable for its use of a regular grid. The methodology described by Conway et al. (*27*) is most similar to the approach of this paper. The authors model the spatial environment with a high-resolution grid, use the ranged-RAPTOR algorithm for path finding, incorporate full schedules, and use accurate walking paths. However, because of their focus on access to jobs, the measurement of value for reaching a given area emphasizes the expected behaviors of commuters.

METHODOLOGY

The purpose of this study is to construct a measurement of a transit network's capability to provide Spontaneous Accessibility: the ability of individuals to make unanticipated trips to unexpected destinations. The measurement is designed as comparable and dimensionless, allowing the evaluation of alternatives as well as the contrasting of the accessibility within areas. Accurately modeling the transit network as seen by riders is a critical component of any accessibility study. However, the literature survey reveals that the most advanced modeling has only been used on studies that focus on home to work trips. Studies that consider accessibility to more destinations have lacked the same precision. Therefore, this section discusses the requirements for a Spontaneous Accessibility measurement and how they are fulfilled.

A Spontaneous Accessibility measurement is an isochrone: it measures a proportion of opportunities that can be reached within a fixed duration. In this case, it is difficult to define what opportunities are. Given that riders are taking trips in response to immediate needs, the value of any destination is potentially unknowable in advance. Rather than attempting to evaluate what areas of the map are of high value, all destinations are defined to provide equal opportunity. The measurement applies this same reasoning to selecting starting points for the isochrone. While it is possible to weight more populous areas as having greater value, such a measurement devalues the burden placed on an individual who finds that they must start their trip from an uncommon origin, and thus are not the default. The measurement also considers every minute of the entire day equally. Increasing the value given to times when the most people travel assumes that unanticipated events occur more often during these periods. The resulting measurement, though expressible as a single dimensionless quantity, is somewhat complex: it convolves every origin point, destination point, and time of day. The use of a strict isochrone, rather than a decay-based measurement, helps limit complexity. Eschewing this dimension preserves "interpretability and communicability", which Geurs and van Wee (*13*) assert is essential for a practical accessibility measurement.

Spatial Environment

An important consideration in designing a Spontaneous Accessibility measurement is appropriately modeling the space that is being evaluated. Unanticipated trips, by their definition, may have origin or destination points anywhere. Practically, however, a Spontaneous Accessibility calculation is made over a bounded area. Furthermore, it would be impossible to determine paths between every point in the area. Therefore, the bounds are divided into uniformly-sized, non-overlapping Sectors. While prior studies have used census or property divisions, their lack of uniformity is problematic. Using non-uniform divisions overestimates the accessibility of areas of the map within large Sectors, causing the measurement to systematically undervalue transit that serves these areas.

Constructing the spatial environment can be done with a very small collection of resources. GTFS files of the transit agencies operating in the region are required to model bus service. For the available walking routes, OpenStreetMap (*28*) data is used. However, the route-finding component is entirely separable, allowing substitution of commercial data sources if available and desired. Water body data, in GeoJSON format (*29*), is used to eliminate entirely-water Sectors. The ease of collecting these resources minimizes the preparation needed to make a Spontaneous Accessibility measurement, enabling it to be performed frequently.

Path Finding

Path finding is used to determine which Sectors fall within the isochrone of a given duration, time of day, and center location. The software preprocesses transit data to support the data access patterns used by the path finder component. A single run of a transit vehicle is called a Trip and is a list of pairs of stops and times. Only Trips that are inside the time span and spatial environment are considered. Additionally, a table of Entry Points has rows of stops and sorted columns of times, allowing reference of the Trip arriving at that stop, at that time.

Walks can be initiated from the starting point or a transit stop and can end at another transit stop or the border of any Sector. All points outside of the bounds are eliminated. To further cut down the number of walking trips that must later be tested for feasibility, the straight-line distance of each potential walking trip is calculated and converted into an estimated time. All estimates less than the maximum duration are retained and sorted by the estimated time. When more precise distances are later needed, the destinations to test are limited to those where the estimated time is less than the allowed time. Final measurement of these distances is done by a separate subsystem, currently an instance of the GraphHopper software (*30*), though commercial products can be substituted.

The path finder itself is intended to provide a highly accurate model of trips that the transit network permits. The algorithm that it uses is a dynamic programming algorithm similar to RAPTOR (*8*) that takes advantage of the preprocessed data. The least-time path is always found; no preference is given to minimizing initial waiting time, waiting time during transfers, or walking distance, though the number of mode transfers is minimized as a consequence of the algorithm. While these properties may conflict with rider preferences, they are intended to describe the network's capability.

Measurement

Every Spontaneous Accessibility measurement is determined by finding paths for a set of Tasks. A Task contains the parameters for performing path finding: a starting time, starting location, and isochrone threshold. Executing a Task yields the Sectors that were reached given the Task's parameters and the best path to each Sector. Consequently, each Sector maintains a Task count: the number of Tasks wherein that Sector was reached. A single executed Task allows the computation of the simplest measurement known as the Time-Qualified Point Accessibility Ratio (TQPAR). This is the ratio of reached Sectors to the total number of Sectors. The results of Anderson et al. (*22*) indicate that accessibility can vary considerably, even within a restricted range of times. Thus, Tasks are executed for every minute of an entire day and the ratios derived from each Task are averaged to create the Point Accessibility Ratio (PAR). The path finder uses techniques similar to ranged-RAPTOR (*8*), to compute these Tasks more efficiently than running them individually. Executing Tasks for the product of every minute of the day and each Sector center, accounts for transit riders taking trips that originate from locations that they cannot anticipate. The ratios from each Task are averaged to give the Network Accessibility Ratio (NAR). Formal descriptions of each calculation are given in Equation 1 where T is the set of all times, S is the set of valid Sectors,

s_0 is a chosen starting point, t_0 is a chosen starting time, and *reached* is a function that computes the number of Sectors reached in a fixed duration for a center point and starting time.

$$\begin{aligned}
TQPAR^{time}_{duration} &= \frac{reached(t_0, s_0)}{|S|} \\
PAR_{duration} &= \frac{\sum\limits_{t \in T} reached(t, s_0)}{|T| \cdot |S|} \\
NAR_{duration} &= \frac{\sum\limits_{t \in T} \sum\limits_{s \in S} reached(t, s)}{|T| \cdot |S|^2}
\end{aligned} \tag{1}$$

Measuring Sampling Error With Kullback-Leibler Divergence

Reducing the number of Tasks comprising a Network Accessibility Ratio provides an opportunity to measure Spontaneous Accessibility at a considerably lower cost. The theoretical validity of this approach is intuitive. Measuring the Spontaneous Accessibility from the center of one Sector also reveals information about nearby Sectors, as the close proximity makes it likely that some of the same transit stations can be reached at similar times of day. Thus, it may be possible to use a sample of Sector centers as starting points rather than every one. Before accepting this approach as valid, it is useful to quantify the impact of this sampling.

In the discipline of information theory, Kullback and Leibler (*31*) describe a technique for measuring the divergence between two random variables where one is an approximation and one is known to be true. The formulation of this, for a true distribution p and an assumed one q, is presented in Equation 2. Using such a technique on Spontaneous Accessibility measurements would require viewing them as random variables.

$$D(p||q) = \sum_{x \in X} p(x) \log \frac{p(x)}{q(x)} \tag{2}$$

While each of the Spontaneous Accessibility ratio measurements yields a single value, they are formulated from what can be thought of as a collection of observations. Thus, given a set of Sectors and their Task counts, it is possible to construct a probability distribution out of the likelihoods of observing Task counts. To allow comparing non-sampled and sampled results, where Task counts will be lower, the ratio of a Sector's Task count to the total number of Tasks is used. These ratios are divided into bins, and the bins must be defined such that every bin with values in the true distribution also has values in the sampled distribution. Otherwise, the Kullback-Leibler Divergence will be infinite, preventing reasonable attempts to compare samplings. Using this strategy, a Network Accessibility calculation and an empirically chosen number of bins is used to generate the distribution for p while the Sampled Network Accessibility and the same number of bins yield q.

Since measuring the divergence from a distribution requires knowing that true distribution, sampling is not used for measuring the present Spontaneous Accessibility of a transit network. However, once a sample with an acceptably low Kullback-Leibler Divergence has been found, planners can test their desired changes using only the sampled points as starting points rather than the centers of every Sector. When calculated with a sample of origins rather than every Sector center, the Spontaneous Accessibility measurement is known as a Sampled Network Accessibility Ratio (SNAR), presented in Equation 3, where S' is the set of sampled Sectors. By performing the SNAR calculation rather than a full NAR, the computation time is reduced, allowing more experimentation.

$$SNAR_{duration} = \frac{\sum\limits_{s \in S'} \sum\limits_{t \in T} reached(t,s)}{|T| \cdot |S| \cdot |S'|} \tag{3}$$

APPLICATION AND RESULTS

To demonstrate the properties and uses of Spontaneous Accessibility measurements, this paper presents two analyses. The first highlights a practical process that planners can use to make decisions using Spontaneous Accessibility. In it, Network Accessibility Ratios quantify the impact of transit network changes. Specifically, it considers one year's worth of transit changes, a light rail extension and bus network restructures, and evaluates whether these changes have been successful in improving the ability to make unexpected trips within Seattle. Planners can use this process to examine historic changes in their transit network. With small modifications, they can use it to evaluate the impact of proposed changes or speculated improvements. The second analysis pertains to the nature of Sampled Network Accessibility Ratios. Though planners are unlikely to replicate this analysis, its results reveal advisable practices when sampling. As such, it is of use to planners who want to test several modifications to the transit network, but are operating under time constraints.

Environment

Seattle is served by three transit agencies: King County Metro operates all-day bus service within the city bounds as well as some commute-focused suburban buses, Sound Transit is a multi-county agency that provides the high-frequency Link light rail line as well as some commuter buses, and the Seattle Department of Transportation manages two streetcar lines. These agencies make service changes at six month intervals. In March of 2016, Sound Transit opened an extension to the Link, with stations in the Capitol Hill neighborhood and near the University of Washington. As a result, King County Metro restructured bus service to eliminate redundancies and provide expanded access to the new light rail stations as part of its six-month periodic restructure process. Many bus lines in neighborhoods surrounding the light rail stations received higher frequencies as a result of the restructure, at the cost of eliminating service considered to be redundant.

Schedule data is available through King County Metro's GTFS files. Though three different agencies control transit in Seattle, King County Metro operates all of the Seattle Department of Transportations streetcars, the Link light rail, and most of the Sound Transit buses that provide value for moving within Seattle. As such, they publish GTFS files including this infrastructure, allowing the use of King County Metro's files alone. Because the analyses span multiple service changes, two sets of GTFS files are needed as these files typically represent service over a fixed date range. Planners doing historic analyses can use this approach; those conducting speculative analysis can construct modified GTFS files reflecting the proposed network changes.

In these analyses, Seattle is physically represented by a bounding rectangle of the city's borders. As a consequence, some areas outside of the actual city boundary are included; for the purposes of this study, they are considered to be a part of Seattle. In their analyses, planners may choose bounds that reflect the totality of their transit network or choose smaller bounds, such as an individual city or neighborhood, if they wish to measure Spontaneous Accessibility for individuals traveling between points in that area. The bounding box is divided into a grid of one hundred by one hundred Sectors. This number is chosen arbitrarily; higher counts more accurately model the experience of individual riders as the smaller Sectors convolve the experiences of fewer individuals, but increase the computational power required to conduct an analysis. In this case each Sector has dimensions of 176 meters by 281 meters (577 feet by 921 feet). These are slightly larger than the typical census blocks in the area, but smaller than all but the smallest census block groups. Sectors entirely on water are eliminated, leaving 6,063 Sectors reachable.

Geographic data of Seattle is freely available. An OpenStreetMap extract of Washington State provides routing data for the entire bounding box. To detect water, a hydrographic map of the region is freely available from the county and was reprojected and converted to GeoJSON with open-source software.

Analyses

To perform a comparative Spontaneous Accessibility analysis of the impact of the year's network changes, Network Accessibility Ratios are selected as the measurement type. This choice reflects that the goal of the analysis is to judge impact of the transit changes to Seattle as a whole. In most all transit planning cases, this is an appropriate measurement, though planners evaluating the impact of transit network modifications on a particular location would compare Point Accessibility Ratios centered at the location. In this retrospective analysis, comparable dates must be chosen. Two non-holiday Mondays at approximately the same time of year, 25 January 2016 and 30 January 2017, represent service before and after the transit network changes took place, while controlling for seasonal variability. A time span of an entire day is required to account for the unpredictability of needs arising. Planners may choose an isochrone duration arbitrarily, but it should reflect data or perception of the amount of time individuals will travel to fulfill an unexpected need. These analyses use a thirty-minute duration.

To evaluate the efficacy of using Sampled Network Accessibility in place of Network Accessibility, several sampling variants are contrasted using otherwise the same parameters as the 25 January 2016 Network Accessibility measurement. This study chose sample sizes of 1,000, 2,000, 3,000, and 4,000 to reflect considerable differences in the amount of sampling used. Five different random samples at each size allow a basic analysis of the distribution of sample quality. For computing the Kullback-Leibler Divergence, planners must empirically choose a range and bin number such that none of the bins from the sampled distribution have zero items. After computing the NAR and the 20 SNARs, it was found that a range from zero to 0.2 with twelve bins fulfilled this requirement.

Discussion

Table 2 summarizes the results of both analyses. The Network Accessibility Ratio measurement from 25 January 2016 of 0.06017 is fairly abstract on its own. It indicates that if an independent selection of a random origin point, destination Sector, and starting time is made, there is a 6.017% chance that that selection will correspond to a trip that can be made in 30 minutes. The value of this number is considerably clarified when viewed comparatively. As a result of the year's transportation changes, the Network Accessibility Ratio on 30 January 2017 was 0.06140. This represents a

TABLE 2 Summary of Spontaneous Accessibility Measurements

Date	Measurement	Samples	Value	K-L Divergence (bits)
2016-01-25	NAR_{30}	6063 (all)	0.06017	0
2016-01-25	$SNAR_{30}$	1000	0.06009	0.00442
2016-01-25	$SNAR_{30}$	1000	0.06080	0.01407
2016-01-25	$SNAR_{30}$	1000	0.06053	0.00155
2016-01-25	$SNAR_{30}$	1000	0.05945	0.00692
2016-01-25	$SNAR_{30}$	1000	0.06109	0.00863
2016-01-25	$SNAR_{30}$	2000	0.05922	0.00420
2016-01-25	$SNAR_{30}$	2000	0.06057	0.00527
2016-01-25	$SNAR_{30}$	2000	0.05987	0.00192
2016-01-25	$SNAR_{30}$	2000	0.05992	0.00219
2016-01-25	$SNAR_{30}$	2000	0.05884	0.00641
2016-01-25	$SNAR_{30}$	3000	0.06050	0.00081
2016-01-25	$SNAR_{30}$	3000	0.06006	0.00197
2016-01-25	$SNAR_{30}$	3000	0.06017	0.00120
2016-01-25	$SNAR_{30}$	3000	0.05980	0.00089
2016-01-25	$SNAR_{30}$	3000	0.05977	0.00196
2016-01-25	$SNAR_{30}$	4000	0.06039	0.00195
2016-01-25	$SNAR_{30}$	4000	0.06013	0.00063
2016-01-25	$SNAR_{30}$	4000	0.05986	0.00171
2016-01-25	$SNAR_{30}$	4000	0.06006	0.00136
2016-01-25	$SNAR_{30}$	4000	0.06018	0.00022
2017-01-30	NAR_{30}	6063 (all)	0.06140	0

2.0% increase in the number of trips with random origin, destination, and starting time combinations that can be made in 30 minutes. The absolute value of this increase is small, but it is taken over the entire bounds of the city at all times of day and corresponds to approximately 65,109,250 additional feasible trips. The relatively small change in value also reflects that the major network changes were localized to the light rail stations and bus service restructures in limited areas. Nevertheless, a planner asked to ascertain whether historical changes have improved the network can use the comparative result to determine that they have increased Spontaneous Accessibility overall. The same process can be employed for comparing several hypothetical alternatives and, before putting one into effect, choosing one with the greatest increase in Network Accessibility Ratio.

In addition to making single-value measurements, planners can spatially decompose Network Accessibility measurements for insight into the distribution of Spontaneous Accessibility. Figure 1 shows the map for the 25 January 2016 Network Accessibility Ratio calculation. Each Sector is colored according to the proportion of Tasks in which that Sector can be reached within 30 minutes. The average of these ratios equals the Network Accessibility Ratio. The map indicates that several patterns of service can create areas with elevated Spontaneous Accessibility. Many bus routes converge on the Central Business District (A), which is also served by light rail. Light rail stations (B, C, D, E, F, G) show pockets of increased Spontaneous Accessibility, though the extent

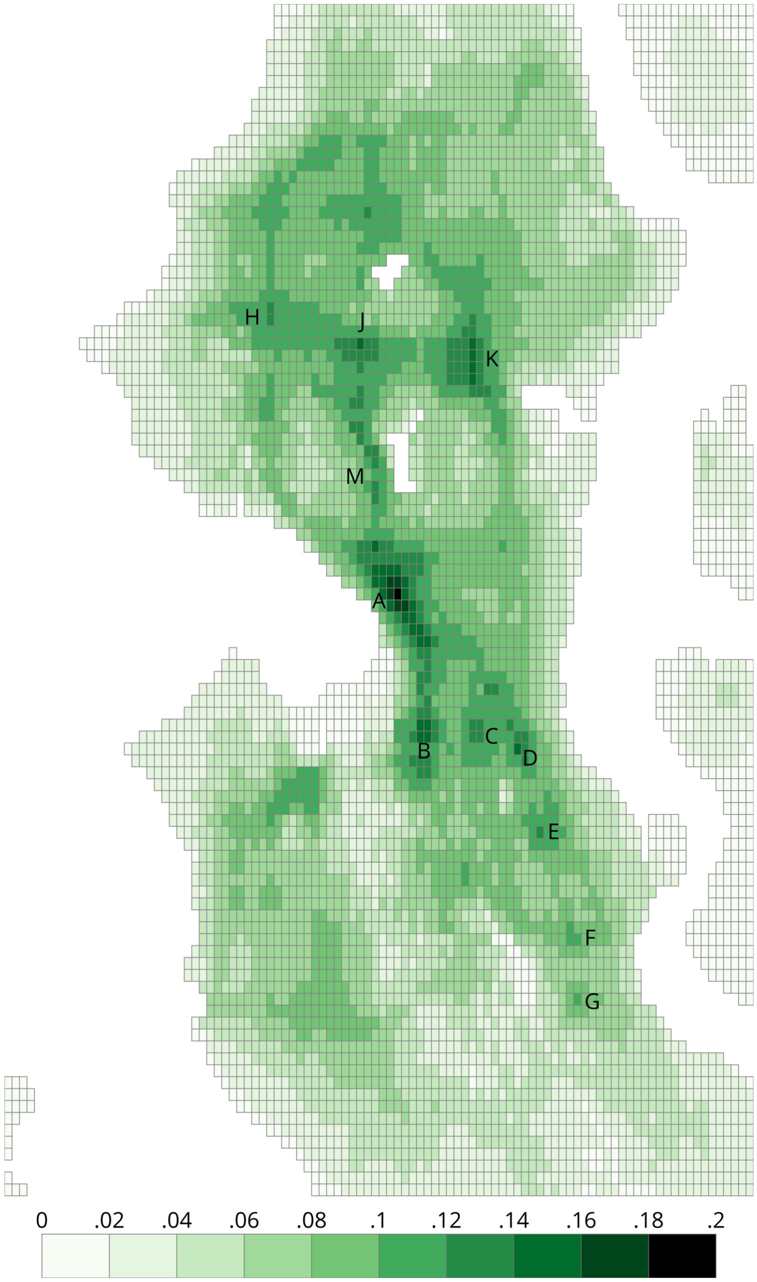

FIGURE 1 This map shows Network Accessibility in a graphical form. Each Sector is colored based on the proportion of Tasks that allow it to be reached within a 30 minute duration. Areas labeled with letters are described in the text.

varies, as a result of the amount of connecting bus service. Areas where frequent buses converge from perpendicular directions (H, J), or where several frequent bus lines converge on a common street (K, M) have accessibilities that match or exceed those of light rail stations outside of downtown. Such a map can also indicate to planners areas for future transit expansion by highlighting areas with low present Spontaneous Accessibility.

With the Network Accessibility map serving as a baseline, Figure 2 shows the impact of

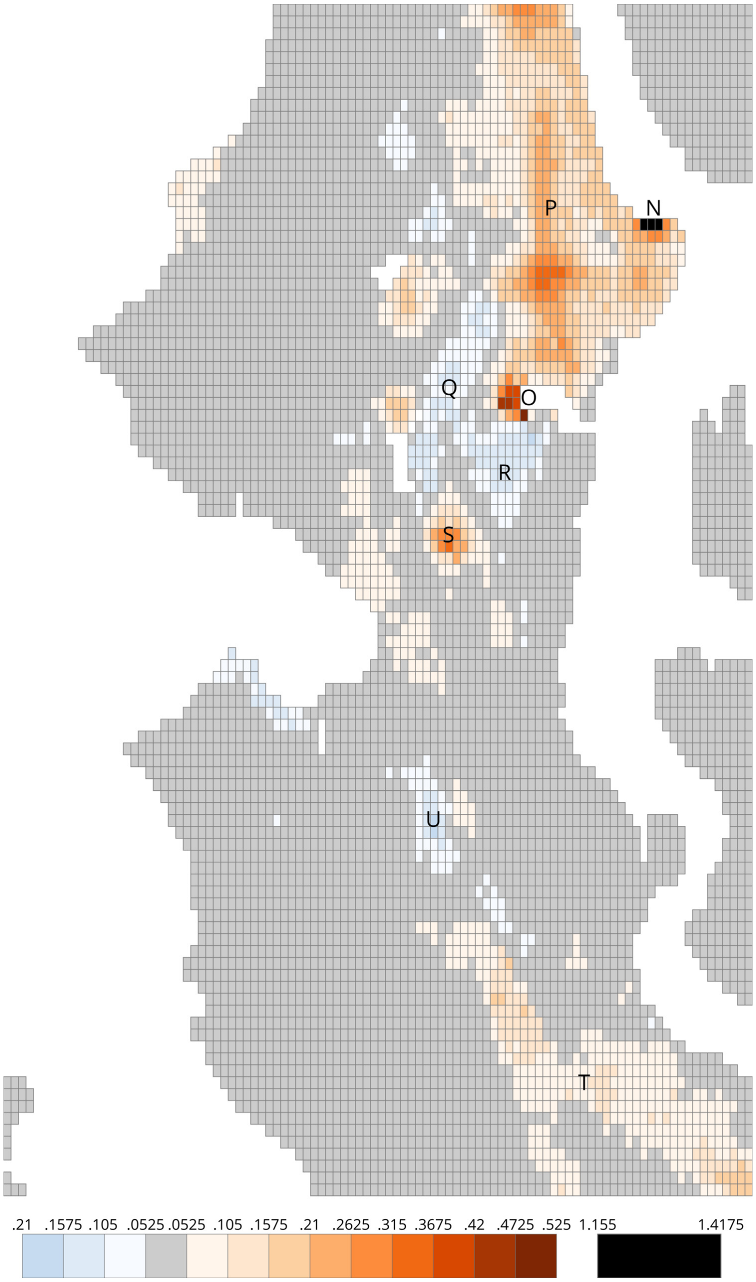

FIGURE 2 This map shows comparative Network Accessibility in a graphical form. Each Sector is colored based on the ratio of change, between 25 January 2016 and 30 January 2017, in the number of Tasks that allow the Sector to be reached within 30 minutes . Sectors more strongly orange are reachable under more circumstances, those more strongly blue under fewer. Areas labeled with letters are described in the text.

the changes made between 25 January 2016 and 30 January 2017. Sectors are colored based on the ratio of improvement or degradation between the earlier and later measurement. The greatest improvement is seen at the terminal of a new frequent bus route where only limited service was available in the past (N). The University of Washington light rail station (O) shows a considerable accessibility increase in its immediate vicinity. While restructured bus service connecting to the station has made unanticipated trips to northeast Seattle (P) more viable, restructures to the west (Q) and south (R) of the station have not been successful in this regard, perhaps the result of more journeys taking an indirect path through the light rail station. Capitol Hill (S) shows noted improvement, but not to the extent of the University of Washington station and northeast Seattle. Bus restructures in this area were limited. In southeast Seattle (T) a large swath of modest improvement is the result of an extension of a bus line through the area. This came at a cost to Georgetown (U), where service through the neighborhood remains equally frequent, but originates from fewer other locations. Viewing the map provides a more nuanced view of the 2.0% gain in Spontaneous Accessibility. A planner can use such a map to qualify the success of a network change, in this case being able to report that benefit has been realized unevenly despite the overall improvement. Furthermore, bus restructures, intended only to reduce redundancy, hurt Spontaneous Accessibility in some areas.

The spatial decomposition of the comparative Network Accessibility also allows planners to construct customized measurements. Though Spontaneous Accessibility is intended to model trips with arbitrary origin and destination points, planners managing limited resources may wish to prioritize service in areas that have historically generated many trips. In this case, planners can weigh the amount of change at individual Sectors by a factor, such as relative measures of ridership, population density, or jobs, before averaging the amount of change. More simply, proposed changes that impact Spontaneous Accessibility negatively for a given set of vital Sectors can be rejected on those grounds. Using decomposed measurements in conjunction with the overall Network Accessibility change can balance planners' goals of improving the network for unexpected trips while maintaining its suitability for existing, predictable ones.

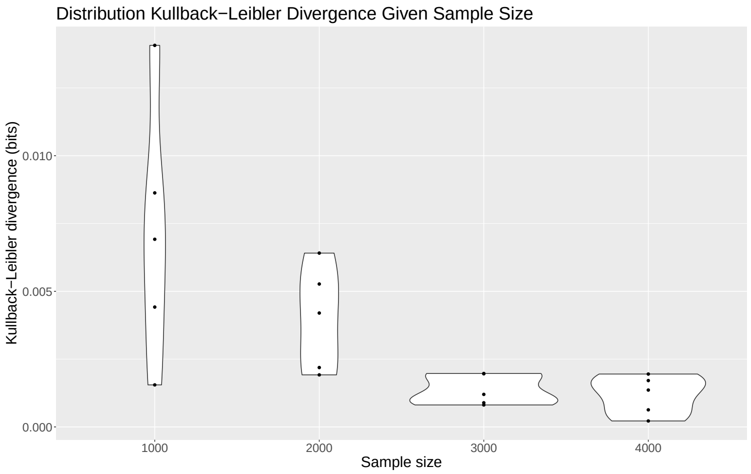

FIGURE 3 Violin plot showing the distribution of Kullback-Leibler divergence given the number of random Sectors with which to calculate the Sampled Network Accessibility.

The results of the second analysis give planners guidance on using Sampled Network Accessibility measurements. Figure 3 graphs the distribution of the Kullback-Leibler divergence of each of five samples at each sampling level. Kullback-Leibler divergence in all cases ranges from zero to one bits of information, but lacks an intuitive sense of scale for making value judgments of the quality of a distribution. For it to be of value, it is used comparatively. This analysis does not find a number of samples beyond which no benefit is gained from adding more. However, the primary benefit of increasing the sample count is lower variability, not necessarily a better sample. The lowest of the 1000-Sector sample rivals the means of the 3,000- and 4,000-Sector samples and has a value lower than any of the 2,000-Sector samples. This result demonstrates that the efficacy of a Sampled Network Accessibility for approximating the Network Accessibility has a strong dependency on Sector selection. Producing a random sample of Sectors and measuring the Kullback-Leibler divergence is a computationally inexpensive operation compared to a Sampled Network Accessibility calculation. The result of this analysis suggests that a time-effective way to compare many network changes is to take several samples at a relatively low sample count and use the sample with the lowest Kullback-Leibler divergence regardless of its absolute value. Confirming this suggestion by examining the distributions of divergence using a different location or alternative Sector sizes is a topic of further research.

The results also suggest that there is value in finding sampling techniques that select a small number of Sectors in a way that minimizes the Kullback-Leibler divergence. This would allow drastically faster analysis of transportation network changes and thus more experimentation. Intuitively, such an algorithm would avoid the selection of Sectors where the reachability of such Sectors can be inferred from adjacent Sectors. A Sector selector that prioritizes Sectors distant from already-selected ones accomplishes this to some extent, but does not account for Sectors that differ from their neighbors considerably for geographic reasons. From a theoretical perspective, Variational Autoencoders (*32*) may provide a way to find such samples. This is a potential topic of future research.

CONCLUSION

By extending the concepts presented in a variety of previous accessibility-based studies of transit networks, this paper builds precise measurements for Spontaneous Accessibility: the ability to make unanticipated trips using public transit. Network Accessibility captures the all-day, full-network accessibility of a public transit system. Its ability to make precise statements about incremental changes such as the Link extension and bus restructures in Seattle demonstrates that it possesses the granularity to be a part of periodic evaluations of a transit network, rather than being saved for long-term planning. Though this study analyzes a transit network change that occurred in the past, planners can use a similar process to evaluate transit network modifications that they may be considering. When planners judge the merits of several alternatives, Sampled Network Accessibility can reduce their time spent by allowing less computationally demanding but still sufficiently representative evaluations. Spontaneous Accessibility measurements are not a wholesale replacement for existing transit planning technologies: they do not address vehicle capacity or incorporate observations of real riders. However, a measurement that encodes the ability of transit customers to make unanticipated trips has value as a supplement, as contemporary research has not emphasized it. By incorporating Spontaneous Accessibility into their planning process, planners can design transit networks that help those who do not have personal vehicles enjoy the advantages of those who do.

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